ELIZADE UNIVERSITY, ILARA-MOKIN, ONDO STATE

FACULTY OF ENGINEERING

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

SEMESTER II EXAMINATION, 2016/2017 ACADEMIC SESSION

COURSE TITLE: NUMERICAL METHODS

COURSE CODE: EEE 312

EXAMINATION DATE: 3rd AUGUST 2017

COURSE LECTURER: DR R. O. Alli-Oke

HOD's SIGNATURE

TIME ALLOWED: 3 HRS

INSTRUCTIONS:

1. ANSWER **QUESTION 1** AND ANY OTHER FOUR QUESTIONS **(TOTAL OF 5 QUESTIONS)**
2. SEVERE PENALTIES APPLY FOR MISCONDUCT, CHEATING, POSSESSION OF UNAUTHORIZED MATERIALS DURING EXAM.
3. YOU ARE **NOT** ALLOWED TO BORROW CALCULATORS AND ANY OTHER WRITING MATERIALS DURING THE EXAMINATION.
4. STATE CLEARLY THE COMBINED STOPPING CONDITIONS USED IN YOUR SOLUTIONS

Question #1

a) The "complex factorization theorem" states that every polynomial that is not identically zero has exactly n (real or complex) roots (counting multiplicity). Prove that every polynomial having real coefficients can be factored into a product of linear and quadratic factors with real coefficients. *Hint: You may use the complex conjugate-root theorem.* **(4 marks)**

b) With the aid of diagrams, clearly compare and contrast between the following numerical methods: Bisection Method, Newton-Raphson method, Regula-Falsi Method, and Secant Method. **(8 marks)**

c) Compute a real root (correct to 4 decimal places) of $f(x) = x^2 - 3$. Error tolerances are 0.01 and start with interval [1.65, 1.75]. Show your workings for only the first iteration. Let a, b denote the end-points and $c = (a+b)/2$ is the bisection point. The tabled results should display the following - $n,\ a, b,\ f(a), f(b),\ c,\ f(c),\ f(a).f(c),\ |b - a|$ **(8 marks)**

Question #2

a) Define the interpolation problem. **(5 marks).**

b) Using Lagrange Basis, determine an approximate function,

$$p(x) = \sum_{i=1}^{n} a_i L_i(x) = \sum_{i=1}^{n} a_i \prod_{\substack{j=1 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j},$$

that interpolates the following data points,

x_i	0	1	−1	2	−2
f_i	−5	−3	−15	39	−9

(5 marks)

Question #3

a) State 3 bases for constructing interpolating polynomials. **(2 marks)**

b) Differentiate between Newton-Cotes quadrature rules and Gaussian quadrature rules. **(2 marks)**

c) Let $f(x) = 3x + \ln x - e^x$. Given that $x = 2.0$ is an approximate root of $f(x)$. Use Newton-Raphson method in determining a real-root of $f(x)$. Take error tolerances of 0.05. **(6 marks)**

Question #4

a) State clearly, the differences between an under-determined, and an over-determined system of linear equations? **(2-marks)**

b) Briefly state the differences between Jacobi method and Gauss-Seidel method? **(2 marks)**

c) Suppose that matrix A can be reduced to row-echelon form using Naïve Gaussian elimination. Show that matrix A has a unique LU decomposition given by,

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix}$$

where l_{21} are the multipliers in Naïve Gaussian elimination process and U is the row echelon of matrix A **(6 marks)**

Question #5

a) Clearly explain the Bairstow's method. **(3 marks)**

b) Compare and contrast the numerical methods for solving system of linear equations **(2 marks)**

c) Given a system of linear equations, $\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} x = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$. Derive the Jacobi method in matrix form,

$$x^{k+1} = -D^{-1}Rx^{k} + D^{-1}b = -D^{-1}(L+U)x^{k} + D^{-1}b$$

where D, L, U are a diagonal matrix, lower-triangular matrix, and an upper-triangular matrix respectively **(5 marks)**

Question #6

a) Let L, U be a unit-diagonal lower-triangular matrix, and an upper-triangular matrix respectively. Let $A = \begin{pmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{pmatrix}$. Use method of LU factorization without pivoting to obtain the LU decomposition for the given A-matrix. **(4 marks)**

b) Use LU factorization to obtain the solution to $\begin{pmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{pmatrix} x = \begin{pmatrix} 26 \\ 0 \\ 0 \end{pmatrix}$ **(3 marks)**

c) Show that a given a 3×3 matrix A admits a $(D + L + U)$ decomposition where D, L, U are a diagonal matrix, unit-diagonal lower-triangular matrix, and an upper-triangular matrix respectively. **(3 marks)**

Question #7

a) What is polynomial division **(1 mark)** and state two types of polynomial division you know **(2 marks)**

b) Use Newton-Raphson method to find the intersection point of the curves $(x-2)^2 - (y-2)^2 = 0$ and $x^2 \sin x + (y-3)^2 = 5$. Show your workings for only the first iteration. Choose an initial point of $(0, 0)$ and error tolerances of 0.05. **(7 marks)**